

THE MODEL OF THE RELATIONSHIP FROM THE BANKS AND INSURANCE COMPANIES

Four levels of the relationship from the banks and insurance companies were studied in the work and for each type of report is proposed the mathematical model: building the conditions necessary for creating the financial balance in insurance companies and banks.

Рассматриваются четыре типа возможного сотрудничества между банком и страховой компанией и для каждого типа отношений строится математическая модель: находятся необходимые условия для создания финансового баланса между банком и страховой компанией.

Розглядаються чотири можливих типи співпраці між банком та страховою компанією і для кожного типу відносин будується математична модель: знаходяться необхідні умови для побудови фінансового балансу між банком і страховою компанією.

Key words: bancassurance, insurance rate, load of risk, parameter of attitude to risk, cooperative e non cooperative games.

At first the term "bancassurance" appeared in France in the late of 60's and it meant the sale of insurance services through the banking network [1]. Today this term is used to intensify the involvement of banks in the industry and marketing for spread of insurance products in the banking sector.

The concept of bancassurance can be described as integration from banks and insurance companies for coordinate the sale amalgamate the banking and insurance products, to enter into the customer base and also have access to the financial resources of partner.

The actuality of this report has the following reasons:

1. The bank has the possibility to:

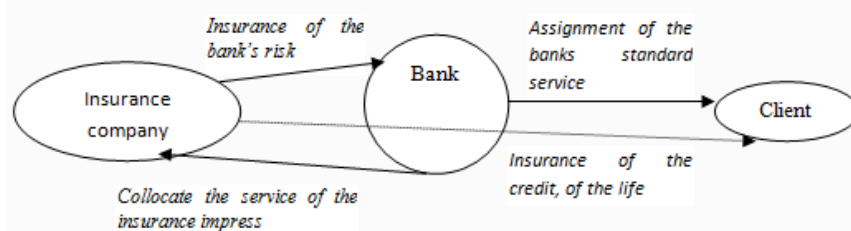
- attracting external resources;
- receiving the commissions from the sale of the policies insurance;
- bettering satisfier the needs of the fixed clients;
- expanding the list of services offered and receive additional benefits for the advancement of own programs;
- forming positive image of the bank;

2. Insurance companies have the possibility to:

- using the data base of the bank's clients;
- reducing the own expenses;
- diffusion the insurance products [3], [5].

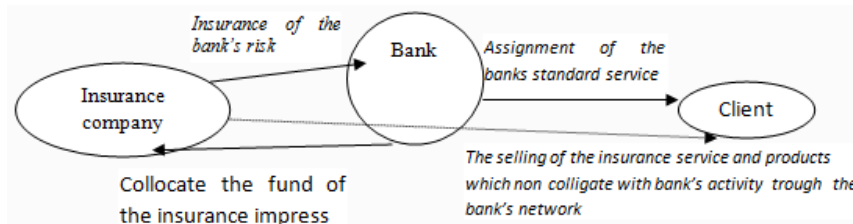
In Italy integration from the banks and insurance companies has begun in the 90's and had a rapid development. Today, the big banking groups have control of insurance companies. Some small banks are controlled by the insurance companies. The products offered by insurance company are the insurance of life and insurance from damage [4]. The character of the integration from banks and insurance companies can be presented in four levels.

1.Basic level. This is simply granting of insurance services to bank's clients. These can be insurance for credit or insurance of life. The collaborations of this type make it possible to insure the risks of banks and collocate on the bank's account the resources of insurance companies.



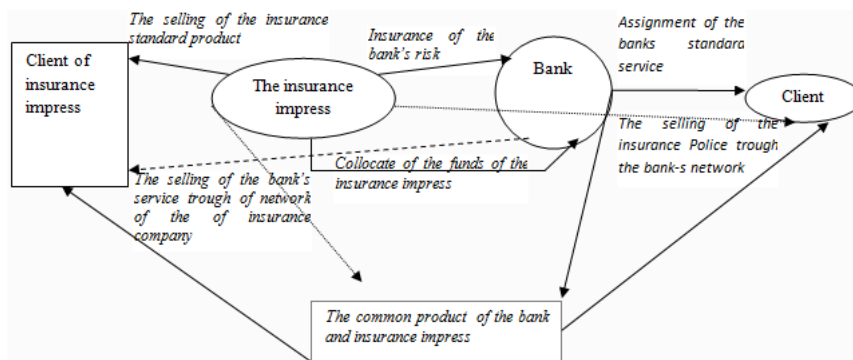
Picture 1. Basic level

2. Average level. It consists of the signing of a relationship which allows spreading the insurance



Picture 2. Average level

3. High level. It is propose of the possibility to create an alliance for cooperation in all fields. In this case it becomes possible to create a common product and diffuse them through a common channel of selling.



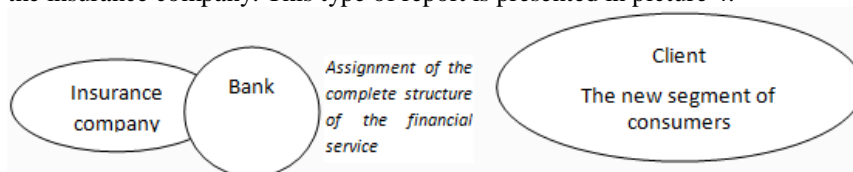
Picture 3. High level

There are positive and negative sides in all type of integration from banks and insurance companies. The negative side can create the problems. The first problem is the instability that comes from conflicts of interest and their incompatibility in various aspects. Other problems are given by different cultures, formations and prospects. There is also the problem of the risks. For insurance companies is important to provide the some eventual compensation of their clients. It's a technical risk for them and it base on the fact that the report from the each party can be solved in another different way from that established in contract. For the banks the principal risk are risk of credit and market and it connects with the investments of the technical reserves using financial character [2], [4]. The risk can be devised in two groups. The first group is formed by the risks analyzed and it can be insurance risk or risk of credit. The second group is formed by the risks non-analyzed. Exactly these propose to integrate the bank in the insurance sector.

There are two ways to manage risk.

- reduce the possibility of damage (techniques stimulation, taxation, increase of reserves and etc...);
- spread risks for reduce the negative consequences of the damage.

4. Higher level. Here is proposed to create a complete connection from the bank and insurance company, or creating an institute of finance assimilate with one another, which becomes the higher level of integration from the bank and the insurance company. This type of report is presented in picture 4.



Picture 4. Higher level

For example, as a common product may be the bank's card with the information of members. The insurance company defends the bank's cards from falsification and proposes new additional services to their clients. The credit

card also can be integrated by insurance service, for example, may be proposed a policy of life that protects the client during a trip abroad.

In Italy, the integration from banks and insurance companies began in the 90's and had a rapid development. Today, the more important banking groups have control of the insurance company. Some banks that have a small size are controlled by the insurance companies. The products they offer are insurance of life and unfortunate situations.

The mathematical models of bancassurance. For each level of the relationship from the banks and insurance companies, which have been studied, we construct a mathematical model and analyze it from the point of view of balance.

The conditions necessary for creating the financial balance in insurance companies are:

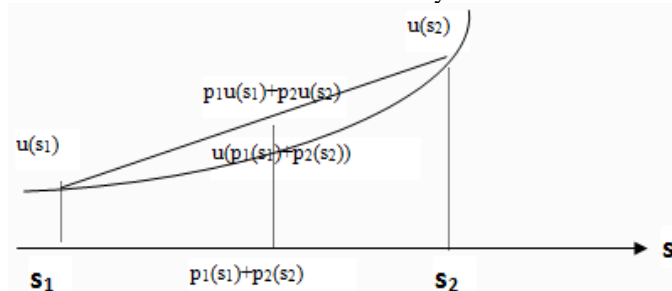
- the sum of the insurance payment must guarantee the eventual insurance reimbursement and provide for the profit of the company;
- the reserves must be sufficient for the insurance compensation, considering the probability of the risks [7].

Now we study the process of building of the insurance rate. The insurance rate r consist from clear premium r_0 and load R . The clean premium depends on the probability p of the event insurance and on the sum insured h : $r_0 = ph$. Regarding the load l , that consist from the commercial load r_c (the cost for maintaining the insurance company), which depends on h : $r_c = \Delta h$ ($0 < \Delta < 1$) and r_r - the load of risk (the minimal guarantee that ensures the insurance impressed by the ruin) and it's a percentage part of the sum insured $r_r = \rho h$. Then $r = r_0 + r_c + r_r = ph + \Delta h + \rho h = h(p + \Delta + \rho)$. If the sum $p + \Delta$ define as the common load ψ we can write definition of the insurance rate: $r = h(p + \psi)$.

Let s is a maximal sum that a client can deposits to receive a compensation s_0 in the case if damage occurs with probability p . The expected profit is the average: $s^* = ps_0$. There are three categories of persons with regard to the attitude towards risk:

- inclined: the sum $s > ps_0$;
- not inclined: the sum $s < ps_0$;
- neutral: the sum $s = ps_0$

Utility $u(s)$ can be defined as the degree of suitability of the investment s ($0 \leq s \leq s_0$). It's dependence analytic from investment and clean profit. In the case when the subject is a neutral toward risk $u(\sum p_i s_i) = \sum p_i u(s_i)$ and his function u is linear, where $u(\sum p_i s_i)$ - utility of the value determined, $\sum p_i u(s_i)$ - utility expected of a value random. When the subject is not inclined toward the risk his function of utility represents the preference the determined value of the utility at random value, that is, $u(\sum p_i s_i) > \sum p_i u(s_i)$. In this case the function $u(s)$ is concave. If $u(\sum p_i s_i) < \sum p_i u(s_i)$ it's said that the subject is inclined toward risk and its function of utility is convex.



Picture 5.

The picture 5 presents the function of utility of the subject that is inclined towards the risk.

Now we introduce "the concept of the parameter ζ of attitude to risk": This is a report from possible waste u_- and possible profit u_+ : $\zeta = u_- / u_+$.

1. The subject is neutral: $s_0 = ps$: $\zeta = 0$.

2. The subject is not inclined: ($s_0 < ps$), the value minimal of s_0 is 0, that is $\zeta = -p \geq -1 \Rightarrow -1 \leq \zeta < 0$.

3. The subject is inclined ($s_0 > ps$): $\zeta = (s - ps) / s = 1 - p \leq 1 \Rightarrow 0 < \zeta \leq 1$

The insurance company is interested in reducing the probability of occurrence of an undesired event and stimulates his clients to give information more exact. This stimulation is called "the bonus for risk" and is calculated using the formula: $\Delta h = h\zeta$, where h is the standard compensation, ζ is the parameter of the attitude to risk of the insurer. So an insurance compensation can be calculated using this formula

$$h = h + \Delta h = h(1 + \zeta). \quad (1)$$

The bank also tries to ensure its risks, that is distributing them. As has been provide [6], the distribution of the risks is impossible in the case when the insurer is neutral ones and then the bank can be inclined or not to risk. Insurance companies usually are neutral by the risk. The bank's clients, who sign the contract with the impressed insurance may be neutral or not inclined to risk.

At the basic level, the bank offers its clients and its services to insurance impresses. In this case the bank, the insurance company and bank's clients are not inclined to risk.

At the average level (Picture 2) the bank offers its clients and funds for the sale of insurance services through its channels and therefore may be inclined to risk.

Advanced level (Picture 3) shows a heterogeneous insurance. The clients of the bank and insurance impress are not inclined to risk when the bank and the insurance sector are inclined.

The contract from the insurance impress and client is a set of strategies of both. A contract is optimal if:

1. The profit of the impress insurance has a maximum value;
2. Signing the contract, the insurer receives a minimum guarantee of usefulness.

There are the commercial insurance and no. If insurance is commercial the profit must be positive, otherwise it is zero. In the present study we analyze the problem of commercial insurance.

For the bank the set of its strategies is: $XB=(x1B, x2B)$, where $x1B$ correspond to the conclusion of a contract with the insurance company, $x2B$ - no, the set of the profit is: $YB=(y1B, y2B)$, where $y1B$ is the compensation that the bank receives choosing the action xiB

For the bank's clients the set of their alternatives is: $YC=(x1C, x2C)$, where $x1C$ correspond to the conclusion of a contract with the insurance company, $x2C$ - no, set of the profits is: $YC=(y1C, y2C)$, where $y1C$ is the benefit that the client receives in the case of choosing the action xiB .

For insurance impress $XA=\{xiA\}$, $i=1,5$, where:

$x1A$ - the parameters of the contract $h=const$ and $r=const$;

$x2A$ - the parameters of the contract: $\psi = const$ and $r_j=h(p_j+\psi)$ (the rate depends on the probability of occurrence of an accident and isn't const);

$x3A$ - the parameters of the contract: the insurance load $r_j=h(p_j+\psi_j)$, then $\psi \neq const$, $r \neq const$;

$x4A$ - commercial insurance;

$x5A$ - non-commercial insurance.

The actions of the insurance company can be described with the pair of elements of the set XA : (xiA, xjA) ($i=1,2,3$; $j=4,5$). But as in this study speaks of insurance commercial that XA : $(xiA, x4A)$ ($i=1,2,3$). The set of profit is $YA=\{yiA\}$, $i=1,4$.

Also there is nature, which can change the result of the profit for each participant. It may be favourable or not: $\theta=(\theta1, \theta2)$ are parameters of the nature of undesirable event. Agents with their actions can correct (reduce) the probability of occurrence of damage. These are prognosis which warn about the insurance situation.

At the basic level (Pic. 1) we have a non-cooperative game. There is the centre (the impress insurance) and $N+1$ agents (N are bank's clients, which can sign the contract with the insurance company).

The insurance contract in this model can be described using the set $\{h, r, FA; xB, xCi, pB, pCi\}$, where h, r, FA are the parameters of the centre (the parameters of the insurance mechanism), xB, xCi, pB, pCi - the parameters of the agents. A contract profitable is a composition of numbers $\{h, r, FA\}$, which makes insurance advantageous for all participants.

Problem 1: It's choose optimal system of stimulation (form the insurance payment and compensation), which gives the maximum profit for all participants.

We study the case when the set of strategies is $X=(x1B; x1C1, \dots, x1Cn; x1A, x4A; \theta2, \dots)$. The insurance company establish the same parameters of the contract for all participants. The, compensation and insurance payment are

$$h=\alpha H, r=\beta h (\alpha > 0, \beta > 0), \quad (3)$$

where H is the profit of the agent in unfavourable situation.

Usefulness of the bank is

$$FB=H_B+p_B(h_B-Q_B)-c_B-r_B, \quad (4)$$

where H_B is the benefit that the bank receives collaboration with its clients (H_{cl}) and the insurance company (H_{as}), c_B are expense for bank's clients, r_B - funds paid for insurance service, p_B is the probability of occurring an accident, h_B - compensation, the insurance company gives the bank in the case of an insured event, Q_B - the loss in the case of a risk.

Signing the contract with the insurance company, the bank's client receives advantage

$$FC_i=H_i+p_i(h_i-Q_i)-c_i-r_i, \quad (5)$$

where H_i is the profit from collaboration with the bank, there is the costs of banking services, r_i - insurance payment, p_i is the probability of an event insured, h_i - compensation in case of accident, Q_i - the losses in the case of a damage.

For insurance company

$$FA=r_B+\sum_{i=1}^n r_i-(p_B h_B+\sum_{i=1}^n p_i h_i). \quad (6)$$

For the bank, the collaboration is an advantageously, when the cost of collaboration don't exceed the compensation which can be received from the insurance company.

$$r_B \leq p_B h_B. \quad (7)$$

$$H_B - c_B - r_B > 0.$$

The bank's client can't pay insurance payment, if it is higher than the compensation.

$$r_i \leq p_i h_i, \forall i.$$

$$H_i - c_i - r_i > 0. \quad (8)$$

For the insurance company its fund may not be less than that required to pay to its clients

$$p_B h_B + \sum_{i=1}^n p_i h_i < r_B + \sum_{i=1}^n r_i. \quad (9)$$

The insurance system also shouldn't encourage the client to be an insured event

$$h_i \leq H_i. \quad (10)$$

Then (4) - (6) can be rewritten

$$\begin{aligned} FB &= H_B + p_B(\alpha H_B - Q_B) - c_B - \alpha \beta H_B; \\ FC_i &= H_i + p_i(\alpha H_i - Q_i) - c_i - \alpha \beta H_i; \\ FA &= \alpha \beta H_B + \sum_{i=1}^n \alpha \beta H_i - (\alpha H_B p_B + \sum_{i=1}^n \alpha H_i p_i). \end{aligned} \quad (11)$$

The conditions (6) - (9) we rewrite

$$\begin{aligned} \alpha &\leq 1; \\ \alpha \beta &\leq 1 - c_i/H_i; \\ \beta &\leq \min\{p_i\}; \\ \beta &\geq (H_B p_B + \sum_{i=1}^n p_i H_i) / (H_B + \sum_{i=1}^n H_i). \end{aligned} \quad (12)$$

Now we can calculate the profit received by each agent in the case of unfortunate event (92): the profit y_{1B} with the strategy x_{1B} : $y_{1B} = H_B + p_B(\alpha H_B - Q_B) - c_B - \alpha \beta H_B$. If the bank chooses the strategy x_{2B} : $y_{2B} = H_B - p_B Q_B - c_B$. The difference from profit in the case of signing the contract or not is

$$\Delta y_B = y_{1B} - y_{2B} = \alpha H_B (p_B - \beta). \quad (13)$$

For bank's customers

$$\Delta y_{iC} = y_{1iC} - y_{2iC} = \alpha H_i (p_i - \beta). \quad (14)$$

For the insurance company the advantage of signing the contract is

$$FA = \alpha H_B (\beta - p_B) - \alpha \sum_{i=1}^n H_i (p_i - \beta) < 0. \quad (15)$$

So we got that for the insurance company the contract with these parameters isn't advantage.

Now the centre takes the strategy x_{2A} and the set of strategies is presented $X = (x_{1B}; x_{1C1}, \dots, x_{1Cn}; x_{2A}, x_{4A}; 92)$.

As has been signed the insurance rate consists from the probability p and the load ψ : $r_i = h(p_i + \psi)$. With regard to compensation, it is equal to the standard h , which the insurance company sets for each client, and the load $h\zeta_i$.

Then (4) - (6) is rewritten:

for bank

$$FB = H_B + p_B(h_B(1 + \zeta_B) - Q_B) - c_B - (p_B + \psi)h_B, \quad (16)$$

for bank's clients

$$FC_i = H_i + p_i(h(1 + \zeta_i) - Q_i) - c_i - (p_i + \psi)h, \quad (17)$$

for insurance company

$$FA = (p_B + \psi)h_B + \sum_{i=1}^n (p_i + \psi)h - ((h_B + \zeta_B)p_B + \sum_{i=1}^n p_i(h_i + \zeta_i)). \quad (18)$$

In the case if the bank refuses to sign the contract with the insurance company, it chooses the strategy x_{2B} , utilities for the bank and its customers are:

$$FB = H_B - p_B Q_B - c_B, \quad (19)$$

$$FC_i = H_i - p_i Q_i - c_i. \quad (20)$$

The rate and compensation for each bank's clients depends by the probability of occurrence an incident. The condition (7) - (11) can be rewrite

$$r_B \leq p_B h_B (1 + \zeta_B), \quad (21)$$

$$r_i \leq p_i h_i (1 + \zeta_i), \quad (22)$$

$$p_B h_B (1 + \zeta_B) + \sum_{i=1}^n p_i h_i (1 + \zeta_i) < r_B + \sum_{i=1}^n r_i. \quad (23)$$

But as $r_B = (p_B + \psi)h_B$ and $r_i = (p_i + \psi)h_i$ we have

$$0 < \psi \leq p_i \zeta_i \leq p_B \zeta_B \Rightarrow \psi \leq \min\{p_i \zeta_i\}.$$

So the load of the insurance rate depends by the parameter of risk of the bank and its clients.

Now we add another condition: stimulation to prevent of occurrence of damage

$$p_B h_B (1 + \zeta_B) \leq Q_B p_B; \quad (24)$$

$$p_i h_i (1 + \zeta_i) \leq Q_i p_i. \quad (25)$$

In case if the insurance company guarantees full compensation for damage the equations (24) - (25) can be rewritten:

$$\begin{aligned} h_B &= Q_B / (1 + \zeta_B), \\ r_B &= Q_B (p_B + \psi) / (1 + \zeta_B), \end{aligned} \quad (26)$$

$$h_i = Q_i / (1 + \zeta_i),$$

$$r_i = Q_i(p_i + \psi) / (1 + \zeta_i)$$

From (26) we can see that insurance payment increases with increase of the probability of the event insurance and increases of loss Q of the client or the bank in case if the above happens. The insurance compensation doesn't depend by the insurance load ψ , but has the linear dependence by the loss and has the non-linear dependence by the parameter of attitude towards risk ζ .

If we replace (24) - (25) in (14) - (16) we the utility can be rewritten

$$FB = H_B - c_B - Q_B(p_B + \psi) / (1 + \zeta_B) \rightarrow \max, \quad (26)$$

$$FC_i = H_i - c_i - Q_i(p_i + \psi) / (1 + \zeta_i) \rightarrow \max, \quad (27)$$

$$FA = Q_B \psi / (1 + \zeta_B) + \sum_{i=1}^n Q_i(\psi - \zeta_i) / (1 + \zeta_i) \rightarrow \max. \quad (28)$$

For the insurance company to sign a contract with the bank has a sense if the load on the insurance rate is higher than the parameter of the attitude towards risk ζ_i for each bank's client ($\psi > \zeta_i$).

If we do subtraction (26) from (17) and (27) from (18) we get

$$\Delta FB = Q_B(p_B \zeta_B - \psi) / (1 + \zeta_B);$$

$$\Delta FC_i = Q_i(p_i \zeta_i - \psi) / (1 + \zeta_i).$$

$$\Delta y_B = y_{1B} - y_{2B} = h_B(p_B \zeta_B - \psi);$$

$$\Delta y_{Ci} = h_i(p_i \zeta_i - \psi);$$

$$y_A = FA.$$

Deduction: in this work were studied several types of the relationship from the bank and insurance company and for each type of report is proposed the mathematical model: were building the conditions necessary for creating the financial balance in insurance companies and banks. Was proposed optimal system of stimulation (form the insurance payment and compensation), which gives the maximum profit for all participants.

Bibliography

1. **Goldman S.** Bancassurance /S. Goldman // European Financial Fact Sheet. L., 2000 – No 7.
2. **Ruozzi R.** Problemi e prospettive della banca – assicurazione in Italia /R. Ruozzi // Il risparmio. 1996 – Vol. 44, No 3.
3. **Раговская А.** Страхование жизни заемщика – гарантия возврата кредита / А. Раговская // Банковский вестник. – К., 2006. – С. 59–60; (..... Insurance the life of borrower is a guarantee of return of the credit. – Kiev, 2006. – P. 59-60).
4. **Desiata A.** Le nuove strategie dell'assicurazione e la bancassurance / A. Desiata // Bancaria. 1994. – Vol. 50, No 5.
5. **Desario V.** Il risparmio finanziario in Italia: strumenti, intermediari, mercati / V. Desario // New Economy e riflessi sui sistemi finanziari e assicurativi, Banca d'Italia, Bollettino Economico, 2001 – No 36.
6. **Бурков В. Н.** Механизмы страхования в социально-экономических системах. / В.Н. Бурков, А.Ю. Заложнев, О.С. Кулик. – М.: ИПУ РАН, 2001. – 109 с.
7. **Robbins E.** Ranking agencies under moral hazard / E. Robbins, B. Sarath // Economic Theory. – 1998. – Vol. 11. – P. 129–156.

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